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INTERVAL VALUED MEMBERSHIP & NON- MEMBERSHIP FUNCTIONS OF INTUITIONISTIC FUZZY P-IDEALS IN BCI-ALGEBRAS

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Abstract: *The purpose of this paper is to define the notion of an interval valued Intuitionistic Fuzzy P-ideal (briefly, an i-v IF P-ideal) of a BCI – algebras. Necessary and sufficient conditions for an i-v Intuitionistic Fuzzy P-ideal are stated. Cartesian product of i-v Fuzzy ideals. Union and intersection of Intuitionistic Fuzzy P-ideals of BCI-algebras are discussed.*

Keywords: BCI-algebra, P-ideal, i-v intuitionistic fuzzy P-ideals, Union and Intersection of i-v intuitionistic fuzzy P-ideals.

1. Introduction

The notion of BCK-algebras was proposed by Imai and Iseki in 1996. In the same year, Iseki [6] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK/BCI-algebras and their relationship with other universal structures including lattices and Boolean algebras. Fuzzy sets were initiated by Zadeh [10]. In [9], Zadeh made an extension of the concept of a Fuzzy set by an interval-valued fuzzy set. This interval-valued fuzzy set is referred to as an i-v fuzzy set. InZadeh also constructed a method of approximate inference using his i-v fuzzy sets. In Birwa's defined interval valued fuzzy subgroups of Rosenfeld`s nature, and investigated some elementary properties. The idea of "intuitionistic fuzzy set" was first published by Atanassov as a generalization of notion of fuzzy sets. After that many researchers considers the Fuzzifications of ideal and sub algebras in BCK/BCI-algebras. In this paper, using the notion of interval valued fuzzy set, we introduce the concept of an interval-valued intuitionistic fuzzy BCI-algebra of a BCI-algebra, and study some of their properties. Using an i-v level set of i-v intuitionistic fuzzy set, we state a characterization of an intuitionistic fuzzy P-ideal of BCI-algebra. We prove that every intuitionistic fuzzy P-ideal of a BCI-algebra X can be realized as an i-v level P-ideal of an i-v intuitionistic fuzzyP-ideal of X. in connection with the notion of homomorphism, we study how the images and inverse images of i-v intuitionistic fuzzy P-ideal become i-v intuitionistic fuzzy P-ideal.

2. Preliminaries:

Let us recall that an algebra $(X,*,0)$ of type $(2,0)$ is called a BCI-algebra if it satisfies the following conditions:1.$((x*y)*(x*z))*(z*y)=0$,
2.$(x*(x*y))*y=0$,
3.$x*x=0$,
4.$x*y=0$ and $y*x=0$ imply $x=y$, for all $x, y, z \in X$.

In a BCI-algebra, we can define a partial ordering"$\leq$" by $x \leq y$ if and only if $x*y=0$in a BCI-algebra X, the set $M=\{x \in X/0*x=0\}$ is a sub algebra and is called the BCK-part of X. A BCI-algebra X is called proper if $X – M\neq\phi$. otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:
1. $(x*y)*z=(x*z)*y$,
2. $x*0=0$,
3. $x \leq y$ imply $x*z \leq y*z$ and $z*y \leq z*x$,
4. $0*(x*y) = (0*x)*(0*y)$,
5. $0*(x*y) = (0*x)*(0*y)$,
6. $0*(0*(x*y)) =0*(y*x)$,
7. $(x *z)*(y*z) \leq x*y$

An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A= \{<x, \mu_A(x), \upsilon_A(x)>/x\in X\}$,Where the functions $\mu_A : X\rightarrow[0,1]$ and $\upsilon_A: X\rightarrow[0,1]$ denote the degree of the membership and the degree of non-membership of each element $x\in X$ to the set A respectively, and $0\leq \mu_A(x) + \upsilon_A(x) \leq 1$ for all $x\in X$.Such defined objects are studied by many authors and have many interesting applications not only in the mathematics. For the sake of simplicity, we shall use the symbol $A=[\mu_A, \upsilon_A]$ for the intuitionistic fuzzy set $A=\{[\mu_A(x),\upsilon_A(x)]/ x\in X\}$.

Definition 2.1:A non - empty subset I of X is called an ideal of X if it satisfies:
1.$0\in I$,
2.$x*y\in I$ and $y\in I \Rightarrow x\in I$.

Definition 2.2:A fuzzy subset μ of a BCI-algebra X is called a fuzzy ideal of X if it satisfies:
1.$\mu(0)\geq\mu(x)$,
2. $\mu(x)\geq\min \{\mu(x*y),\mu(y)\}$,for all $x, y\in X$.

Definition 2.3: A non-empty subset I of X is called a P- ideal of X if it satisfies:

1.0ϵI.
2. (x*z)*(y*z)ϵI and yϵI imply x*zϵI.Putting z=0 in(2) then we see that every P- ideal is an ideal.
Definition 2.4:A fuzzy set μ in a BCI-algebra X is called an fuzzy P- ideal of X if
1.μ(0)≥μ(x),
2. μ(x)≥min {μ ((x*z)*(y*z)),μ(y)}.
Definition 2.5: An IFS A=< X, μ_A, υ_A> in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:
(F1) μ_A (0)≥ μ_A(x) & $\upsilon_A(0) \geq \upsilon_A(x)$,
(F2) μ_A(x)≥ min {μ_A (x*y), μ_A(y)},
(F3) υ_A (x) ≤ max {υ_A(x*y), υ_A (y)}, for all x, yϵX
Definition 2.6: An intuitionistic fuzzy set A=< μ_A, υ_A> of a BCI-algebra X is called an intuitionistic fuzzy P- ideal if it satisfies (F1) and
(F4) μ_A(x)≥min {μ_A((x*z)*(y*z)), μ_A (y)},
(F5) υ_A(y*x)≤max{υ_A((x*z)*(y*z)), υ_A (y)}, for all x,y,z ϵ X.

An interval-valued intuitionistic fuzzy set A defined on X is given byA={(x,[μ_A^L(x)μ_A^U(x)],[υ_A^L(x)υ_A^U(x)])},$\forall x\epsilon X$ where μ_A^L,μ_A^U are two membership functions and υ_A^L,υ_A^U are two non-membership functions X such that $\mu_A^L \leq \mu_A^U$&$\upsilon_A^L \geq \upsilon_A^U$,∀ x$\epsilon$X. Let $\bar{\mu}_A$(x)=[μ_A^L,μ_A^U]&$\overline{\upsilon_A}$(x)=[υ_A^L,υ_A^U],∀ xϵX and let D[0,1]denote the family of all closed subintervals of [0,1].If μ_A^L(x)=μ_A^U(x)=c,0≤c≤1 and if υ_A^L(x)=υ_A^U(x)=k, 0≤k≤1,then we have $\bar{\mu}_A$(x)=[c,c]&$\bar{\upsilon}_A$(x)=[k,k] which we also assume, for the sake of convenience, to belong to D[0,1]. thus $\bar{\mu}_A$(x)&$\bar{\upsilon}_A$(x)ϵ[0,1],∀ xϵX,and therefore the i-v IFS a is given by A=[(x,$\bar{\mu}_A$(x),$\bar{\upsilon}_A$(x))},∀ xϵX,where $\bar{\mu}_A$(x):X→D[0,1]. Now let us define what is known as refined minimum, refined maximum of two elements in D[0,1].we also define the symbols" ≤","≥" and "=" in the case of two elements in D[0,1]. Consider two elements D_1:[a_1,b_1]and D_2:[a_2,b_2]ϵD[0,1]. Then rmin(D_1,D_2)=[min{a_1,a_2},min{b_1,b_2}],rmax(D_1,D_2)=[max{a_1,a_2},max{b_1,b_2}]$D_1 \geq D_2$⇔ $a_1 \geq a_2$,$b_1 \geq b_2$;$D_1 \leq D_2$⇔ $a_1 \leq a_2$,$b_1 \leq b_2$ and D_1=D_2.

3.Interval-valued Intuitionistic fuzzy P-ideals of BCI-algebras

Definition 3.1: An interval-valued intuitionistic fuzzy set A in BCI-algebra X is called an interval-valued intuitionistic fuzzy P-ideal of X if it satisfies
(FI_1)$\bar{\mu}_A$(0) ≥$\bar{\mu}_A$(x),$\bar{\upsilon}_A$(0) ≤$\bar{\upsilon}_A$(x),
(FI_2)$\bar{\mu}_A$(x)≥r min {$\bar{\mu}_A$((x*z)*(y*z)),$\bar{\mu}_A$(y)},
(FI_3)$\bar{\upsilon}_A$(x) ≤ r max {$\bar{\upsilon}_A$((x*z)*(y*z)),$\bar{\upsilon}_A$(y)}.
Theorem 3.2 Let A be an i-v intuitionistic fuzzy P-ideal of X. if there exists a sequence {x_n} in X such that
$\lim_{n\to\infty}\bar{\mu}_A(x_n)$=[1,1],$\lim_{n\to\infty}\bar{\nu}_A(x_n)$=[0,0] then $\bar{\mu}_A$(0)=[1,1] and$\bar{\upsilon}_A$(0)=[0,0].
Proof: Since $\bar{\mu}_A$(0) ≥$\bar{\mu}_A$(x)and $\bar{\upsilon}_A$(0) ≤$\bar{\upsilon}_A$(x) for all xϵX, we have $\bar{\mu}_A$(0) ≥$\bar{\mu}_A$(x_n) and $\bar{\upsilon}_A$(0) ≤$\bar{\upsilon}_A$(x_n), for every positive integer n. note that$[\mu_A^L,\mu_A^U]\geq\bar{\mu}_A(0)$.[1,1] ≥$\bar{\mu}_A$(x) ≥$\bar{\mu}_A$(0) ≥$\lim_{n\to\infty}\bar{\mu}_A(x_n)$ =[1,1]. $[\lambda_A^L,\lambda_A^U]\leq\bar{\lambda}_A(0)$.[0,0] ≤$\bar{\upsilon}_A$(x)≤ $\bar{\upsilon}_A$(0)≤$\lim_{n\to\infty}\bar{\nu}_A(x_n)$=[0,0].Hence $\bar{\mu}_A$(0)=[1,1] and$\bar{\upsilon}_A$(0)=[0,0].
Lemma3.3: An i-v intuitionistic fuzzy set A= [⟨μ_A^L, μ_A^U⟩,⟨υ_A^L, υ_A^U⟩] in X is an i-v intuitionistic fuzzy P-ideal of X if and only if ⟨μ_A^L, μ_A^U⟩and ⟨υ_A^L, υ_A^U⟩ are intuitionistic fuzzy ideals of X.

Proof: Since $\mu_A^L(0) \geq \mu_A^L(x)$; $\mu_A^U(0) \geq \mu_A^U(x)$; $\upsilon_A^L(0) \leq \upsilon_A^L(x)$ and $\upsilon_A^U(0) \leq \upsilon_A^U(x)$, Therefore $\bar{\mu}_A(0) \geq \bar{\mu}_A(x)$, $\bar{\upsilon}_A(0) \leq \bar{\upsilon}_A(x)$.

Suppose that $\langle \mu_A^L, \mu_A^U \rangle$ and $\langle \upsilon_A^L, \upsilon_A^U \rangle$ are intuitionistic fuzzy ideal of X. let x,yϵX, then

$\bar{\mu}_A(x)=[\mu_A^L(x),\mu_A^U(x)] \geq [\min\{\mu_A^L(x*y),\mu_A^L(y)\},\min\{\mu_A^U(x*y),\mu_A^U(y)\}]$

$=r\min\{[\mu_A^L(x*y), \mu_A^U(x*y)],[\mu_A^L(y), \mu_A^U(y)]\}$

$= r\min\{\bar{\mu}_A(x*y),\bar{\mu}_A(y)\}$ and

$\bar{\upsilon}_A(x)= [\upsilon_A^L(x),\upsilon_A^U(x)] \leq [\max\{\upsilon_A^L(x*y),\upsilon_A^L(y)\},\max\{\upsilon_A^U(x*y),\upsilon_A^U(y)\}]$

$=r\max\{[\upsilon_A^L(x*y), \upsilon(x*y)],[\upsilon_A^L(y), \upsilon(y)]\}$

$= r\max\{\bar{\upsilon}_A(x*y),\bar{\upsilon}_A(y)\}$.

Hence A is an i-v intuitionistic fuzzy ideal of X.

Conversely,

Assume that A is an i-v intuitionistic fuzzy ideal of X. for any x,yϵX,we have

$[\mu_A^L(x),\mu_A^U(x)]= \bar{\mu}A(x) \geq r\min\{[\bar{\mu}_A(x*y),\bar{\mu}_A(y)]\}$

$=r\min\{[\mu_A^L(x*y), \mu_A^U(x*y)],[\mu_A^L(y), \mu_A^U(y)]\}$

$= [\min\{\mu_A^L(x*y),\mu_A^L(y)\},\min\{\mu_A^U(x*y),\mu_A^U(y)\}]$

And $[\upsilon_A^L(x),\upsilon_A^U(x)] =\bar{\upsilon}_A(x) \leq r\max\{\bar{\upsilon}_A(x*y),\bar{\upsilon}_A(y)\}$

$=r\max\{[\upsilon_A^L(x*y), \upsilon_A^U(x*y)],[\upsilon_A^L(y), \upsilon_A^U(y)]\}$

$= [\max\{\upsilon_A^L(x*y),\upsilon_A^L(y)\},\min\{\upsilon_A^U(x*y),\upsilon_A^U(y)\}]$

It follows that $\mu_A^L(x) \geq \min\{\mu_A^L(x*y),\mu_A^L(y)\}$, $\upsilon_A^L(x) \leq \max\{\upsilon_A^L(x*y),\upsilon_A^L(y)\}$

And $\mu_A^U(x) \geq \min\{\mu_A^U(x*y),\mu_A^U(y)\}$, $\upsilon_A^U(x) \leq \max\{\upsilon_A^U(x*y),\upsilon_A^U(y)\}$

Hence $\langle \mu_A^L, \mu_A^U \rangle$ and $\langle \upsilon_A^L, \upsilon_A^U \rangle$ are intuitionistic fuzzy ideals of X.

Theorem 3.4. Every i-v intuitionistic fuzzy P-ideal of a BCI-algebra X is an i-v intuitionistic fuzzy ideal.

Proof: Let $A=[\langle \mu_A^L, \mu_A^U \rangle, \langle \upsilon_A^L, \upsilon_A^U \rangle]$ be an i-v intuitionistic fuzzy P-ideal of X, where $\langle \mu_A^L, \mu_A^U \rangle$ and $\langle \upsilon_A^L, \upsilon_A^U \rangle$ are intuitionistic fuzzy P-ideal of X. thus $\langle \mu_A^L, \mu_A^U \rangle$ and $\langle \upsilon_A^L, \upsilon_A^U \rangle$ are intuitionistic fuzzy P-ideals of X. hence by lemma 3.3, A is i-v intuitionistic fuzzy ideal of X.

Definition 3.5: An i-v intuitionistic fussy set A in X is called an interval-valued intuitionistic fuzzy BCI-sub algebra of X if $\bar{\mu}_A(x*y) \geq r\min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ and $\bar{\upsilon}_A(x*y) \leq \{\bar{\upsilon}_A(x),\bar{\upsilon}_B(y)\}$, for all x, y$\epsilon$X.

Theorem 3.6: Every i-v intuitionistic fuzzy P-ideal of a BCI-algebra X is i-v intuitionistic fuzzy sub algebra of X.

Proof: Let A=[$\langle \mu_A^L, \mu_A^U \rangle$, $\langle \upsilon_A^L, \upsilon_A^U \rangle$]be an i-v intuitionistic fuzzy P-ideal of X, where $\langle \mu_A^L, \mu_A^U \rangle$, and $\langle \upsilon_A^L, \upsilon_A^U \rangle$ are intuitionistic fuzzy P-ideal of BCI-algebra X. thus $\langle \mu_A^L, \mu_A^U \rangle$, and $\langle \upsilon_A^L, \upsilon_A^U \rangle$are intuitionistic fuzzy subalgebra of X. Hence, A is i-v intuitionistic fuzzy sub algebra of X.

4. Cartesian product of i-v intuitionistic fuzzy P-ideals

Definition 4.1: Let$\bar{\mu}_B$,$\bar{\upsilon}_B$ respectively, be an i-v membership and non- membership function of each element xϵX to the set B.Then strongest i-v intuitionistic fuzzy set relationon X ,that is a membership function relation $\bar{\mu}_A$on$\bar{\mu}_B$ and non- membership function relation $\bar{\upsilon}_A$ on$\bar{\upsilon}_B$ and μ_{A_B} , ν_{A_B} whose i-v membership and non- membership function, of each element (x, y) ϵX×X and defined by $\bar{\mu}_{A_B}$ (x, y)=r min{$\bar{\mu}_B$(x),$\bar{\mu}_B$(y)}& $\bar{\nu}_{A_B}$ (x, y)=r max{$\bar{\upsilon}_B$(x),$\bar{\upsilon}_B$(y)}

Definition 4.2: Let B=[$\langle \mu_B^L, \mu_B^U \rangle$, $\langle \upsilon_B^L, \upsilon_B^U \rangle$] be an i-v subset in a set X, then the strongest i-v intuitionistic fuzzy relation on X that is a i-v A on B is A_B and defined by, $A_B = [\langle \mu_{A_B}^L, \mu_{A_B}^U \rangle, \langle \nu_{A_B}^L, \nu_{A_B}^U \rangle]$

Theorem4.3: Let $B = [\langle \mu_{A_B}^L, \mu_{A_B}^U \rangle, \langle \nu_{A_B}^L, \nu_{A_B}^U \rangle]$be an i-v subset in a set X and $A_B = [\langle \mu_{A_B}^L, \mu_{A_B}^U \rangle, \langle \nu_{A_B}^L, \nu_{A_B}^U \rangle]$be the strongest i-v intuitionistic fuzzy relation on X. then B is an i-v intuitionistic P-ideal of X if and only if A_B is an i-v intuitionistic fuzzy P-ideal of X×X.

Proof: Let B be an i-v intuitionistic fuzzy P-ideal of X. then

$\bar{\mu}_{AB}$(0,0)=r min{$\bar{\mu}_B$(0),$\bar{\mu}_B$(0)}

≥ r min{$\bar{\mu}_B$(x),$\bar{\mu}_B$(y)}=$\bar{\mu}_{AB}$(x, y) and $\bar{\upsilon}_{AB}$(0,0)=r max{$\bar{\upsilon}_B$(0),$\bar{\upsilon}_B$(0)}≤r max{$\bar{\upsilon}_B$(x),$\bar{\upsilon}_B$(y)}=$\bar{\upsilon}_{AB}$(x, y) ∀(x, y) ϵX×X.

On the other hand $\bar{\mu}_{A_B}$ (x_1, x_2) = r min {$\bar{\mu}_B(x_1)$,$\bar{\mu}_B(x_2)$}

≥ r min{r min{$\bar{\mu}_B((x_1*z_1)*(y_1*z_1))$,$\bar{\mu}_B(y_1)$},r min{$\bar{\mu}_B((x_2*z_2)*(y_2*z_2))$,$\bar{\mu}_B(y_2)$}}

=r min{r min{$\bar{\mu}_B((x_1*z_1)*(y_1 *z_1))$,$\bar{\mu}_B((x_2*z_2)*(y_2 *z_2))$},r min {$\bar{\mu}_B(y_1)$,$\bar{\mu}_B(y_2)$}}

=r min {$\bar{\mu}_{AB}((x_1*z_1)*(y_1 *z_1), (x_2*z_2)*(y_2 *z_2))$,$\bar{\mu}_{AB}(y_1, y_2)$}

=r min {$\bar{\mu}_{AB}$ (((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_1, z_2))),$\bar{\mu}_{AB}(y_1, y_2)$}

Also, $\bar{\nu}_{A_B}$ (x_1, x_2)=r max {$\bar{\upsilon}_B$ (x_1),$\bar{\upsilon}_B(x_2)$}

≤ r max{r max{$\bar{\upsilon}_B((x_1*z_1)*(y_1 *z_1))$,$\bar{\upsilon}_B(y_1)$},r max{$\bar{\upsilon}_B((x_2*z_2)*(y_2 *z_2))$,$\bar{\upsilon}_B(y_2)$}}

$= r\max\{r\max\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B((x_2*z_2)*(y_2*z_2))\},r\max\{\bar{\upsilon}_B(z_1),\bar{\upsilon}_B(z_2)\}\}$

$=r\max\{\bar{\upsilon}_{AB}((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(z_2*y_2)),\bar{\upsilon}_{AB}(y_1,y_2)\}$

$=r\max\{\bar{\upsilon}_{AB}(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_{1,}z_2))),\bar{\upsilon}_{AB}(z_1,z_2)\}$

For all $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ in X×X. hence A_B is an i-v intuitionistic fuzzy P-ideal of X×X.

Conversely,

let A_B be an i-v intuitionistic fuzzy P-ideal of X×X. then for all $(x, x)\epsilon X\times X$.we have

$r\min\{\bar{\mu}_B(0),\bar{\mu}_B(0)\}=\bar{\mu}_{AB}(0,0)\geq\bar{\mu}_{AB}(x,x) = r\min\{\bar{\mu}_B(x),\bar{\mu}_B(x)\}(or)\bar{\mu}_B(0)\geq\bar{\mu}_B(x)$ and

$r\max\{\bar{\upsilon}_B(0),\bar{\upsilon}_B(0)\}=\bar{\upsilon}_{AB}(0,0)\leq\bar{\upsilon}_{AB}(x,x) = r\min\{\bar{\upsilon}_B(x),\bar{\mu}_B(x)\}(or)\bar{\upsilon}_B(0)\leq\bar{\upsilon}_B(x)\forall x\epsilon X$. Now,

Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\ \epsilon X\times X$, then

$r\min\{\bar{\mu}_B(x_1,x_2)\}=\bar{\mu}_{AB}(x_1,x_2)\geq r\min\{\bar{\mu}_{AB}(((x_1,x_2)*((z_1,z_2))*((y_1,y_2)*(z_1,z_2))),\bar{\mu}_{AB}(y_1,y_2)\}$

$=r\min\{\bar{\mu}_{AB}((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2)),\bar{\mu}_{AB}(y_1,y_2)\}$

$=r\min\{r\min\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\},r\min\{\bar{\mu}_{AB}((x_2*z_2)*(y_2*z_2)),\bar{\mu}_B(y_2)\}\}$

Also, $r\max\{\bar{\upsilon}_B(x_1,x_2)\}=\bar{\upsilon}_{AB}(x_1,x_2)$

$\leq r\max\{\bar{\upsilon}_{AB}(((x_1,x_2)*((z_1,z_2))*((y_1,y_2)*(z_1,z_2))),\bar{\upsilon}_{AB}(y_1,y_2)\}$

$=r\max\{\bar{\upsilon}_{AB}(((x_1*z_1)*(y_1*z_1)),((x_2*z_2)*(y_2*z_2))),\bar{\upsilon}_{AB}(y_1,y_2)\}$

$=r\max\{r\max\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\},r\max\{\bar{\upsilon}_{AB}((x_2*z_2)*(y_2*z_2)),\bar{\upsilon}_B(y_2)\}\}$

If $x_2=y_2=z_2=0$, then

$r\min\{\bar{\mu}_B(x_1),\bar{\mu}_B(0)\}\geq r\min\{r\min\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\},\bar{\mu}_B(0)\}$ and

$r\max\{\bar{\upsilon}_B(x_1),\bar{\upsilon}_B(0)\}\geq r\max\{r\max\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B(y_1)\},\bar{\upsilon}_B(0)\}$

$\bar{\mu}_B(x_1)\geq r\min\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\}$ and

$\bar{\upsilon}_B(x_1)\geq r\max\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B(y_1)\}$.

Therefore B is i-v intuitionistic fuzzy P-ideal of X.

Definition 4.4: An intuitionistic fuzzy relation A on any set a is a intuitionistic fuzzy subset A with a membership function Ω_A: X×X→ [0, 1] and non- membership function Ψ_A: X×X→ [0, 1].

Lemma 4.5: Let $\bar{\mu}_A$and $\bar{\mu}_B$ be two membership functions and $\bar{\upsilon}_A$ and $\bar{\upsilon}_B$ be two non- membership functions of each $x\ \epsilon X$ to the i-v subsets A and B, respectively. Then $\mu_A\times\mu_B$ is membership function and $\upsilon_A\times\upsilon_B$ is non- membership function of each element$(x,y)\epsilon X\times X$ to the set A×B and defined by $(\bar{\mu}_A\times\bar{\mu}_B)(x,y)=r\min\{\bar{\mu}_A(x),\bar{\mu}_B(y)\}$ and

$(\bar{\upsilon}_A\times\bar{\upsilon}_B)(x,y) = r\max\{\bar{\upsilon}_A(x),\bar{\upsilon}_B(y)\}$.

Definition 4.6: Let A= $[\langle \mu_A^L, \mu_A^U \rangle, \langle \upsilon_A^L, \upsilon_A^U \rangle]$ and B=$[\langle \mu_B^L, \mu_B^U \rangle, \langle \upsilon_B^L, \upsilon_B^U \rangle]$ be two i-v intuitionistic fuzzy subsets in a set X. The Cartesian product of A×B is defined by A×B= $\{((x, y), \bar{\mu}_A \times \bar{\mu}_B, \bar{\upsilon}_A \times \bar{\upsilon}_B); \forall x,y \epsilon X\times X\}$Where A×B: X×X→D[0,1].

Theorem 4.7: Let A=$[\langle \mu_A^L, \mu_A^U \rangle, \langle \upsilon_A^L, \upsilon_A^U \rangle]$and B=$[\langle \mu_B^L, \mu_B^U \rangle, \langle \upsilon_B^L, \upsilon_B^U \rangle]$ be two i-v intuitionistic fuzzy subsets in a set X, then A×B is an i-v intuitionistic fuzzy P-ideal of X×X.

Proof: Let(x, y) ϵX×X, then by definition

$(\bar{\mu}_A \times \bar{\mu}_B)(0,0) = r\min\{\bar{\mu}_A(0), \bar{\mu}_B(0)\}$

$= r\min\{[\mu_A^L(0), \mu_A^U(0)], [\mu_B^L(0), \mu_B^U(0)]\}$

$= [\min\{\mu_A^L(0), \mu_B^L(0)\}, \min\{\mu_A^U(0), \mu_B^U(0)\}]$

$\geq [\min\{\mu_A^L(x), \mu_B^L(y)\}, \min\{\mu_A^U(x), \mu_B^U(y)\}]$

$= r\min\{[\mu_A^L(x), \mu_A^U(x)], [\mu_B^L(y), \mu_B^U(y)]\}$

$= r\min\{\bar{\mu}_A(x), \bar{\mu}_B(y)\}$

$= (\bar{\mu}_A \times \bar{\mu}_B)(x, y)$

And $(\bar{\upsilon}_A \times \bar{\upsilon}_B)(0,0) = r\max\{\bar{\upsilon}_A(0), \bar{\upsilon}_B(0)\}$

$= r\max\{[\upsilon_A^L(0), \upsilon_A^U(0)], [\upsilon_B^L(0), \upsilon_B^U(0)]\}$

$= [\max\{\upsilon_A^L(0), \upsilon_B^L(0)\}, \max\{\upsilon_A^U(0), \upsilon_B^U(0)\}]$

$\leq [\max\{\upsilon_A^L(x), \upsilon_B^L(y)\}, \max\{\upsilon_A^U(x), \upsilon_B^U(y)\}]$

$= r\max\{[\upsilon_A^L(x), \upsilon_A^U(x)], [\upsilon_B^L(y), \upsilon_B^U(y)]\}$

$= r\max\{\bar{\upsilon}_A(x), \bar{\upsilon}_B(y)$

$= (\bar{\upsilon}_A \times \bar{\upsilon}_B)(x, y)$

Therefore (FI_2) holds. Now, for all x, y, zϵX, we have

$(\bar{\mu}_A \times \bar{\mu}_B)((x, x')) = r\min\{\mu_A(x), \mu_B(x')\}$

$\geq r\min\{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}, r\min\{\bar{\mu}_A((x^1*z^1)*(y^1*z^1)), \bar{\mu}_A(y^1)\}\}$

$= r\min\{\{\min\{\mu^L{}_A((x*z)*(y*z)), \mu^L{}_A(y)\}, \min\{\mu^U{}_A((x*z)*(y*z)), \mu^U{}_A(y)\}\},$
$\{\min\{\mu^L{}_B((x^1*z^1)*(y^1*z^1)), \mu^L{}_B(y^1)\}, \min\{\mu^U{}_B((x^1*z^1)*(y^1*z^1)), \mu^U{}_B(y^1)\}\}$

$= \{\min\{\min\{\mu^L{}_A((x*z)*(y*z)), \mu^L{}_B((x^1*z^1)*(y^1*z^1))\}, \min\{\mu^L{}_A(y), \mu^L{}_B(y^1)\}\},$
$\min\{\min\{\mu^U{}_A((x*z)*(y*z)), \mu^U{}_B((x^1*z^1)*(y^1*z^1))\}, \min\{\mu^U{}_A(y), \mu^U{}_B(y^1)\}\}\}$

$= r\min\{(\bar{\mu}_A \times \bar{\mu}_B)(((x*z)*(y*z)), ((x^1*z^1)*(y^1*z^1))), (\bar{\mu}_A \times \bar{\mu}_B)(y, y')\}$

Also, $(\bar{\upsilon}_A \times \bar{\upsilon}_B)((x, x')) = r\max\{\upsilon_A(x), \upsilon_B(x')\}$

$$\leq r\max\{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},r\max\{\bar{\nu}_A((x^1*z^1)*(y^1*z^1)),\bar{\nu}_A(y^1)\}\}$$

$$=r\max\{\{\max\{\nu^L{}_A((x*z)*(y*z)),\nu^L{}_A(y)\},\ \max\{\nu^U{}_A((x*z)*(y*z)),\nu^U{}_A(y)\}\},$$
$$\{\max\{\nu^L{}_B((x^1*z^1)*(y^1*z^1)),\nu^L{}_B(y^1)\},\max\{\nu^U{}_B((x^1*z^1)*(y^1*z^1)),\nu^U{}_B(y^1)\}\}$$

$$=\{\max\{\max\{\nu^L{}_A((x*z)*(y*z)),\nu^L{}_B((x^1*z^1)*(y^1*z^1))\},\max\{\nu^L{}_A(y),\ \nu^L{}_B(y^1)\}\},$$
$$\max\{\max\{\nu^U{}_A((x*z)*(y*z)),\nu^U{}_B((x^1*z^1)*(y^1*z^1))\},\max\{\nu^U{}_A(y),\nu^U{}_B(y^1)\}\}\}$$

$$=\ r\ max\ \{(\bar{\nu}_A\times\bar{\nu}_B)(((x*z)*(y*z)),((x^1*z^1)*(y^1*z^1)),(\bar{\nu}_A\times\bar{\nu}_B)(y,y^1)\}$$

Hence A×B is an i-v intuitionistic fuzzy P-ideal of X× X

Definition 4.8: Let A be a fuzzy ideal of BCI algebra X. The fuzzy set A^m with membership function μ_{A^m} is defined by $\mu_{A^m}(x)\leq(\mu_A(x))^m,\forall x\in X$

Theorem 4.9: If $\bar{\mu}_A$ is a i-v intuitionistic fuzzy a-ideal of BCI-algebra X, then $\bar{\mu}_{A^m}$ is also i-v intuitionistic fuzzy P-ideal of BCI-algebra X

Proof: For all x, y, z $\in$ X

1. $\bar{\mu}_A(0)\geq\bar{\mu}_A(x),\ \bar{\nu}_A(0)\leq\bar{\nu}_A(x)$

$\Rightarrow\left[\bar{\mu}_A(0)\right]^m\geq\left[\bar{\mu}_A(x)\right],\left[\bar{\nu}_A(0)\right]^m\leq\left[\bar{\nu}_A(x)\right]$

$\Rightarrow\bar{\mu}_A(0)^m\geq\bar{\mu}_A(x)^m,\ \bar{\nu}_A(0)^m\leq\bar{\nu}_A(x)^m.$

$\Rightarrow\bar{\mu}_{A^m}(0)\geq\bar{\mu}_{A^m}(x),\ \bar{\nu}_{A^m}(0)\leq\bar{\nu}_{A^m}(x),\ \forall x\in X$

2. $\bar{\mu}_A(x)\geq r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\}$

$\Rightarrow\left[\bar{\mu}_A(x)\right]^m\geq\left[r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\}\right]^m$

$\Rightarrow\bar{\mu}_A(x)^m\geq r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\}^m$

$\Rightarrow\bar{\mu}_{A^m}(x)\geq r\min\{\bar{\mu}_A((x*z)*(y*z))^m,\bar{\mu}_A(y)^m\}$

$\Rightarrow\bar{\mu}_{A^m}(x)\geq r\min\{\bar{\mu}_{A^m}((x*z)*(y*z)),\bar{\mu}_{A^m}(y)\}$

3. $\bar{\nu}_A(x)\leq r\max\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\}$

$\Rightarrow\left[\bar{\nu}_A(x)\right]^m\leq\left[r\max\ \{\bar{\nu}_A((x*z)*(y*z))\bar{\nu}_A(y)\}\right]^m$

$\Rightarrow\bar{\nu}_A(x)^m\leq r\max\ \{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\}^m$

$\Rightarrow\bar{\nu}_{A^m}(x)\leq r\max\ \{\bar{\nu}_A((x*z)*(y*z))^m,\ \bar{\nu}_A(y)^m\}$

$\Rightarrow\bar{\nu}_{A^m}(x)\leq r\max\{\bar{\nu}_{A^m}((x*z)*(y*z)),\ \bar{\nu}_{A^m}(y)\}$

5. Union and Intersection of i-v intuitionistic fuzzy P-ideals

Definition 5.1: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cup B$ with membership function $\mu_{A\cup B}$ is defined by $\mu_{A\cup B}(\mathrm{x})=\max\{\mu_A(\mathrm{x}),\mu_B(\mathrm{x})\},\forall x\in X.$

Definition 5.2: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A\cap B$ membership function $\mu_{A\cap B}$ is defined by $\mu_{A\cap B}(\mathrm{x})=\min\{\mu_A(\mathrm{x}),\mu_B(\mathrm{x})\}, x\in X.$

Definition 5.3: Let A and B be two fuzzy ideal of BCI algebra X with membership function and respectively. A is contained in B if $\mu_A(\mathrm{x})\le\mu_B(\mathrm{x}),\ \forall\ x\in X$

Theorem 5.4: If $\bar{\mu}_A$ is a i-v intuitionistic fuzzy P-ideal of BCI-algebra X, then $\bar{\mu}_{A\cup B}$ is also a i-v intuitionistic fuzzy P-ideal of BCI-algebra X.

Proof: For all x, y, z $\in$ X

$$1.\ \bar{\mu}_A(0)\ge\bar{\mu}_A(x),\ \bar{\nu}_A(0)\le\bar{\nu}_A(x)\ and\ \bar{\mu}_B(0)\ge\bar{\mu}_B(x),\ \bar{\nu}_B(0)\le\bar{\nu}_B(x)$$
$$\min\{\bar{\mu}_A(0),\bar{\mu}_B(0)\}\ge\min\{\bar{\mu}_A(x),\bar{\mu}_B(x)\},\ \min\{\bar{\nu}_A(0),\ \bar{\nu}_B(0)\}\le\min\{\bar{\nu}_A(x),\ \bar{\nu}_B(x)\}$$
$$\bar{\mu}_{A\cup B}(0)\ge\bar{\mu}_{A\cup B}(x),\ \bar{\nu}_{A\cup B}(0)\le\bar{\nu}_{A\cup B}(x)$$

$$2.\ \bar{\mu}_A(x)\ge r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},\quad \bar{\mu}_B(x)\ge r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_A(y)\}$$
$$\{\bar{\mu}_A(x),\bar{\mu}_B(x)\}\ge\ \{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},\ r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_B(y)\}\}$$
$$\max\{\bar{\mu}_A(x),\bar{\mu}_B(x)\}\ge\ \max\{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},\ r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_B(y)\}\}$$
$$\ge\ \max\{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_B((x*z)*(y*z))\},\quad r\max\{\bar{\mu}_A(y),\bar{\mu}_B(y)\}\}$$

If one is contained in the other

$$r\min\{\max\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_B((x*z)*(y*z))\},\quad \max\{\bar{\mu}_A(y),\bar{\mu}_B(y)\}\}$$
$$\bar{\mu}_{A\cup B}(x)\ge r\min\{\bar{\mu}_{A\cup B}((x*z)*(y*z)),\bar{\mu}_{A\cup B}(y)\}$$

$$3.\ \bar{\nu}_A(x)\le r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},\quad \bar{\nu}_B(x)\le r\max\{\bar{\nu}_B((x*z)*(y*z)),\bar{\mu}_A(y)\}$$
$$\{\bar{\nu}_A(x),\bar{\nu}_B(x)\}\le\ \{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},\ r\max\{\bar{\nu}_B((x*z)*(y*z)),\bar{\nu}_B(z)\}\}$$
$$\max\{\bar{\nu}_A(x),\bar{\nu}_B(x)\}\le\ \max\{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},\ r\max\{\bar{\nu}_B((x*z)*(y*z)),\bar{\nu}_B(y)\}\}$$
$$\bar{\nu}_{A\cup B}(x)\le\ r\max\{\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_B((x*z)*(y*z))\},\ \max\{\bar{\nu}_A(y),\bar{\nu}_B(y)\}\}$$
$$\bar{\nu}_{A\cup B}(x)\le r\max\{\bar{\nu}_{A\cup B}((x*z)*(y*z)),\bar{\nu}_{A\cup B}(y)\}$$

Theorem 5.5: If $\bar{\mu}_A$ is a i-v intuitionistic fuzzy R-ideal of BCI-algebra X, then $\bar{\mu}_{A\cap B}$ is also a i-v intuitionistic fuzzy P-ideal of BCI-algebra X

Proof: For all x, y, z $\in$ X

$1.\bar{\mu}_A(0)\geq\bar{\mu}_A(x),\ \bar{\nu}_A(0)\leq\bar{\nu}_A(x)\ and\ \bar{\mu}_B(0)\geq\bar{\mu}_B(x),\ \bar{\nu}_B(0)\leq\bar{\nu}_B(x)$

$\min\{\bar{\mu}_A(0),\bar{\mu}_B(0)\}\geq\min\{\bar{\mu}_A(x),\bar{\mu}_B(x)\},\min\{\bar{\nu}_A(0),\ \bar{\nu}_B(0)\}\leq\min\{\bar{\nu}_A(x),\ \bar{\nu}_B(x)\}$

$\bar{\mu}_{A\cap B}(0)\geq\bar{\mu}_{A\cap B}(x),\ \bar{\nu}_{A\cap B}(0)\leq\bar{\nu}_{A\cap B}(x)$

$2.\bar{\mu}_A(x)\geq r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},\bar{\mu}_B(x)\geq r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_A(y)\}$

$\{\bar{\mu}_A(x),\bar{\mu}_B(x)\}\geq\ \{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_B(y)\}\}$

$\min\{\bar{\mu}_A(x),\bar{\mu}_B(x)\}\geq\ \min\{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},r\min\{\bar{\mu}_B((x*z)*(y*z)),\bar{\mu}_B(y)\}\}$

$\geq\ \min\{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_B((x*z)*(y*z))\},r\min\{\bar{\mu}_A(y),\bar{\mu}_B(y)\}\}$

$\bar{\mu}_{A\cap B}(x)\geq r\min\{\bar{\mu}_{A\cap B}((x*z)*(y*z)),\bar{\mu}_{A\cap B}(y)\}$

$3.\bar{\nu}_A(x)\leq r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},\bar{\nu}_B(x)\leq r\max\{\bar{\nu}_B((x*z)*(y*z)),\ \bar{\nu}_A(y)\}$

$\{\bar{\nu}_A(x),\bar{\nu}_B(x)\}\leq\ \{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},\ r\max\{\bar{\nu}_B((x*z)*(y*z)),\bar{\nu}_B(y)\}\}$

If one is contained in the other

$\min\{\bar{\nu}_A(x),\bar{\nu}_B(x)\}\leq\min\{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},r\max\{\bar{\nu}_B((x*z)*(y*z)),\bar{\nu}_B(y)\}\}$

$\bar{\nu}_{A\cap B}(x)\leq\ r\max\{\min\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_B((x*z)*(y*z))\},\min\{\bar{\nu}_A(y),\bar{\nu}_B(y)\}\}$

$\bar{\nu}_{A\cap B}(x)\leq r\max\{\bar{\nu}_{A\cap B}((x*z)*(y*z)),\bar{\nu}_{A\cap B}(y)\}$

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